RUHR-UNIVERSITÄT BOCHUM

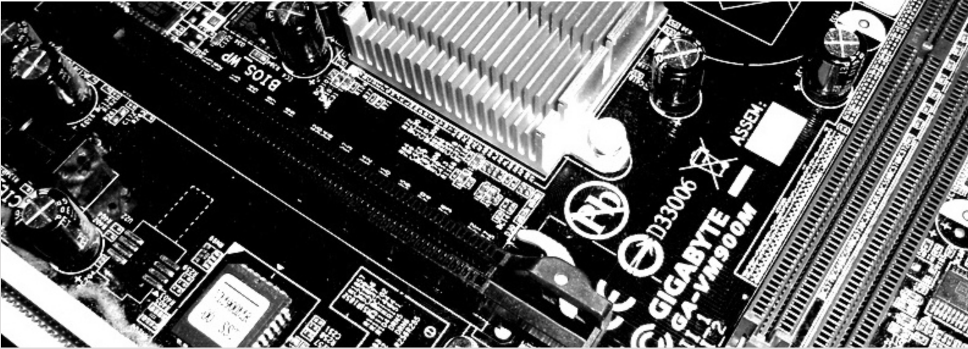
XOR Count

November 21st, 2017

FluxFingers

Workgroup Symmetric Cryptography
Ruhr University Bochum

Friedrich Wiemer



Overview

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Joint Work – Its not me alone [Kra+17][1]

Thorsten Kranz, Gregor Leander, Ko Stoffelen, Friedrich Wiemer

RUHR UNIVERSITÄT BOCHUM RUB

Radboud University

Outline

[1] available on eprint: `https://eprint.iacr.org/2017/1151`

What is the XOR count,
and why is it important?

Some facts

- Lightweight Block Ciphers
- Efficient Linear Layers
- MDS matrices are “optimal” (regarding security)[2]

[2] Are they?

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- MDS matrices are “optimal” (regarding security)[2]
- What is the lightest implementable MDS matrix?
- What about additional features (Involutory)?

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- MDS matrices are “optimal” (regarding security)[2]
- What is the lightest implementable MDS matrix?
- What about additional features (Involutory)?

The XOR count

- Metric for needed hardware resources
- Smaller is better

[2] Are they?

What is an MDS matrix?

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Definition: MDS

A matrix M of dimension k over the field $\mathbb{F}$ is *maximum distance separable* (MDS), iff all possible submatrices of M are invertible (or nonsingular).

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Example

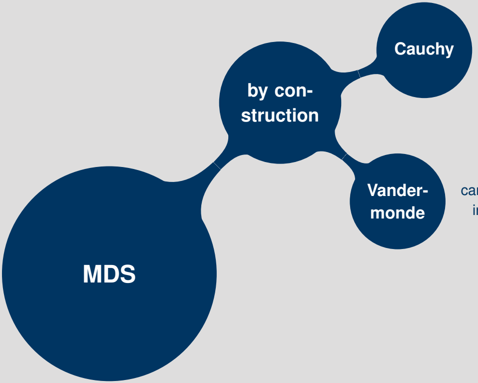
The AES MIXCOLUMN matrix is defined over $\mathbb{F}_{2^8} \cong \mathbb{F}[x]/\texttt{0x11b}$:

$$\begin{pmatrix} \texttt{0x02} & \texttt{0x03} & \texttt{0x01} & \texttt{0x01} \\ \texttt{0x01} & \texttt{0x02} & \texttt{0x03} & \texttt{0x01} \\ \texttt{0x01} & \texttt{0x01} & \texttt{0x02} & \texttt{0x03} \\ \texttt{0x03} & \texttt{0x01} & \texttt{0x01} & \texttt{0x02} \end{pmatrix} = \begin{pmatrix} x & x+1 & 1 & 1 \\ 1 & x & x+1 & 1 \\ 1 & 1 & x & x+1 \\ x+1 & 1 & 1 & x \end{pmatrix}$$

This is a (right) *circulant* matrix: $\operatorname{circ}(x, x+1, 1, 1)$.

What is an MDS matrix?

Constructions



can be made involutory

What is an MDS matrix?

Constructions

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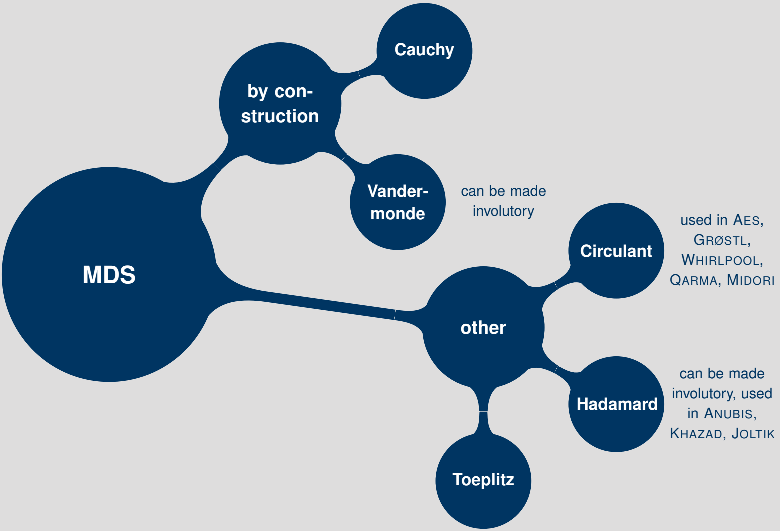

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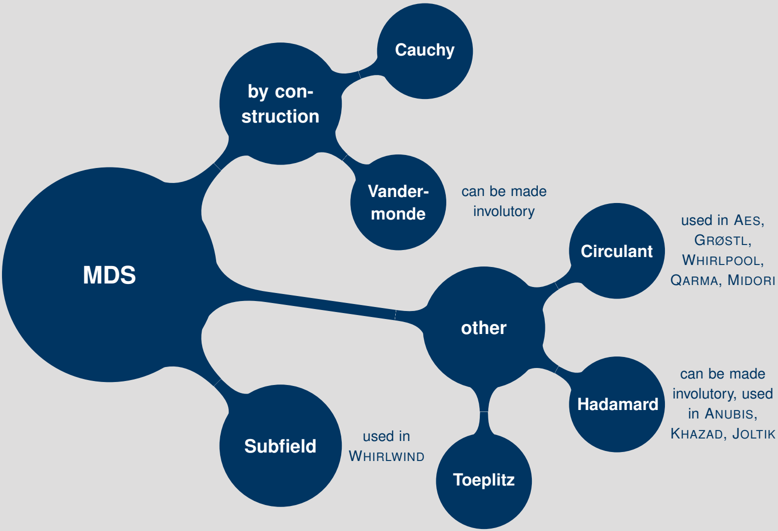

What is an MDS matrix?

Representations

How to implement this in hardware?

- This is about hardware implementations
- How do we implement a field multiplication in hardware?
- How do we implement a matrix multiplication in hardware?

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- How do we implement a matrix multiplication in hardware?

Example

$$\alpha \longrightarrow \boxed{\cdot 1} \longrightarrow \beta \qquad \alpha \longrightarrow \boxed{\cdot x} \longrightarrow \beta \qquad \alpha \longrightarrow \boxed{\cdot (x+1)} \longrightarrow \beta$$

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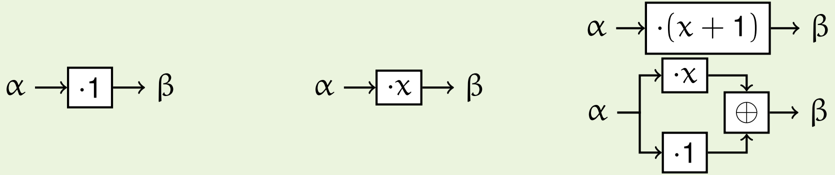
Example



Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to $\mathbb{F}_2^n$

Implement $\alpha \rightarrow \boxed{\cdot 1} \rightarrow \beta$

OK, this one is easy ☺

Example in $\mathbb{F}_2[x]/0x13$:

Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to $\mathbb{F}_2^n$

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Example in $\mathbb{F}_2[x]/0x13$:

$$\alpha = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$$

$$\begin{aligned}\beta &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 \\ &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3\end{aligned}$$

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Example in $\mathbb{F}_2[x]/\text{0x13}$:

$$
\begin{aligned}
\alpha &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 \\
x^4 &\equiv x + 1 \bmod \text{0x13} \\
\beta &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 \\
&= x \cdot (\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3) \\
&\equiv \alpha_3 + (\alpha_0 + \alpha_3) x + \alpha_1 x^2 + \alpha_2 x^3)
\end{aligned}
$$

Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to $\mathbb{F}_2^n$

In matrix notation for $\mathbb{F}_2[x]/0x13$:

$$\beta = 1 \cdot \alpha \Leftrightarrow \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

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Companion Matrix

We call $M_{p(x)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ the *companion matrix* of the polynomial $p(x) = 0x13$. For any element $\gamma \in \mathbb{F}_2[x]/p(x)$, we denote by M_γ the matrix that implements the multiplication by this element in $\mathbb{F}_2^n$.

Counting XOR's

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Example

We can rewrite the AES MIXCOLUMN matrix as:

$$\mathcal{M}_{\text{AES}} = \text{circ}(x, x+1, 1, 1) \cong \text{circ}(M_x, M_{x+1}, M_1, M_1).$$

Starting in $(\mathbb{F}_2[x]/\text{0x11b})^{4\times4}$, we end up in $\left({\mathbb{F}_2}^{8\times8}\right)^{4\times4} \cong \mathbb{F}_2^{32\times32}$.

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Starting in $(\mathbb{F}_2[x]/\text{0x11b})^{4\times4}$, we end up in $\left(\mathbb{F}_2{}^{8\times8}\right)^{4\times4} \cong \mathbb{F}_2^{32\times32}$.

A first XOR-count

To implement multiplication by γ, we need $\text{hw}(M_\gamma) - \dim(M_\gamma)$ many XOR's. Thus

$$\begin{aligned}\text{XOR-count}(\mathcal{M}_{\text{AES}}) &= 4 \cdot (\text{hw}(M_x) + \text{hw}(M_{x+1}) + 2 \cdot \text{hw}(M_1)) - 32 \\ &= 4 \cdot (11 + 19 + 2 \cdot 8) - 32 = 152.\end{aligned}$$

The General Linear Group

Generalise a bit

Instead of choosing elements from $\mathbb{F}_{2^n} \cong \mathbb{F}_2[x]/p(x)$ we can extend our possible choices for “multiplication matrices” by exploiting the following.

The General Linear Group

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Instead of choosing elements from $\mathbb{F}_{2^n} \cong \mathbb{F}_2[x]/p(x)$ we can extend our possible choices for “multiplication matrices” by exploiting the following.

Todo

Maybe remove this?

RUHR-UNIVERSITÄT BOCHUM

The Stupidity of recent XOR Count Papers

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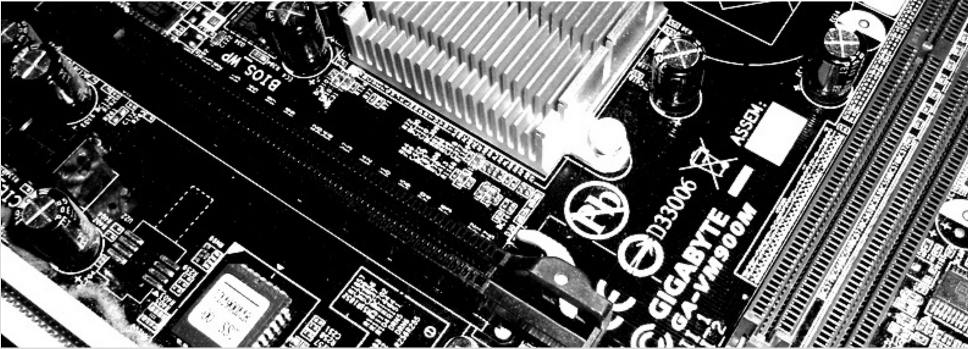

State of the Art

Before our Paper

- You saw how to count XORs
- This count is split in the “overhead” and the XORs needed for the field multiplication
- Thus for AES we get $56 + 8 \cdot 3 \cdot 4 = 56 + 96 = 152$
- Finding a good matrix reduces now to find the cheapest elements for field multiplication
- There is a lot of work followng this line [BKL16; JPS17; LS16; LW16; LW17; Sim+15; SS16a; SS16b; SS17; ZWS17]

State of the Art

Best known Results

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4×4 matrices over $GL(8, \mathbb{F}_2)$		
Matrix	Naive	Literature
AES (Circulant)	152	**7+96**
[Sim+15] (Subfield)	136	40+96
[LS16] (Circulant)	128	32+96
[LW16]	106	10+96
[BKL16] (Circulant)	136	24+96
[SS16b] (Toeplitz)	123	27+96
[JPS17] (Subfield)	122	20+96

Related Work I

Optimized Arithmetic for Reed-Solomon Encoders

Christof Paar*
ECE Department
Worcester Polytechnic Institute
Worcester, MA 01609
email: christof@ece.wpi.edu

1997 IEEE International Symposium on Information Theory, June 29 -- July 4, 1997, Ulm, Germany (extended version)

Abstract

Multiplication with constant elements from Galois fields of characteristic two is the major arithmetic operation in Reed-Solomon encoders. This contribution describes two optimization algorithms which yield low complexity constant multipliers for Ga-

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Related Work II

J. Cryptol. (2013) 26: 280–312
DOI: 10.1007/s00145-012-9124-7

Journal of
CRYPTOLOGY

Logic Minimization Techniques with Applications to Cryptology*

Joan Boyar†
Department of Mathematics and Computer Science, University of Southern Denmark, Odense, Denmark
joan@imada.sdu.dk

Philip Matthews‡
Aarhus University, Aarhus, Denmark

René Peralta
Information Technology Laboratory, NIST, Gaithersburg, MD, USA
rene.peralta@nist.gov

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Received 8 February 2011
Online publication 3 May 2012

State of the Art

Best known Results (After our Paper)

4×4 matrices over $GL(8, \mathbb{F}_2)$

Matrix	Naive	Literature
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State of the Art

Best known Results (After our Paper)

4×4 matrices over $GL(8, \mathbb{F}_2)$					
			Our Results [Kra+17]		
Matrix	Naive	Literature	PAAR1	PAAR2	BP
AES (Circulant)	152	**7+96**	108	108	97
[Sim+15] (Subfield)	136	40+96	100	98	100
[LS16] (Circulant)	128	32+96	116	116	112
[LW16]	106	10+96	102	102	102
[BKL16] (Circulant)	136	24+96	116	112	110
[SS16b] (Toeplitz)	123	27+96	110	108	107
[JPS17] (Subfield)	122	20+96	96	95	**86**

State of the Art

Finding better matrices?

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Type	Previously Best Known		XOR count
$\mathrm{GL}(4, \mathbb{F}_2)^{4\times4}$	58	[JPS17; SS16b]	36
$\mathrm{GL}(8, \mathbb{F}_2)^{4\times4}$	106	[LW16]	72
$(\mathbb{F}_2[x]/\texttt{0x13})^{8\times8}$	392	[Sim+15]	196
$\mathrm{GL}(8, \mathbb{F}_2)^{8\times8}$	640	[LS16]	392
$(\mathbb{F}_2[x]/\texttt{0x13})^{4\times4\star}$	63	[JPS17]	42
$\mathrm{GL}(8, \mathbb{F}_2)^{4\times4}$	126	[JPS17]	84
$(\mathbb{F}_2[x]/\texttt{0x13})^{8\times8}$	424	[Sim+15]	212
$\mathrm{GL}(8, \mathbb{F}_2)^{8\times8}$	663	[JPS17]	424

Future Work; Questions?

Thank you for your attention!

Do your work!

Apply global optimization techniques that are known for years!

(But thanks for the easy paper ☺)

Mainboard & Questionmark Images: flickr

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