

BISON Instantiating the Whitened Swap-Or-Not Construction

EuroCrypt – May 23rd, 2019

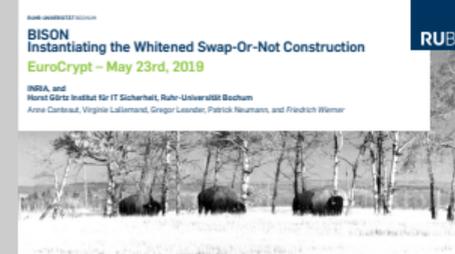
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RUB

2019-05-18

BISON Instantiating the Whitened Swap-Or-Not Construction



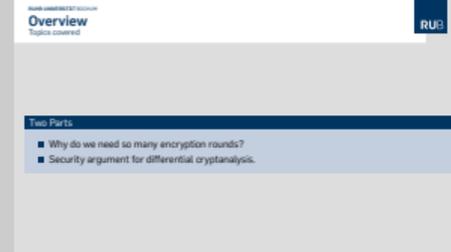
- Whitened Swap-Or-Not Construction developed by Hoang et al. and Tessaro
- Way of building block ciphers
- As this is one of the few talks here at EuroCrypt about block ciphers, we will start simple



Two Parts

- Why do we need so many encryption rounds?
- Security argument for differential cryptanalysis.

- Talk mainly about two parts of the paper:
 - Why do we need so many rounds
(easy to understand argument)
 - Security against differential cryptanalysis
(again relative simple argument that gives strong security here)

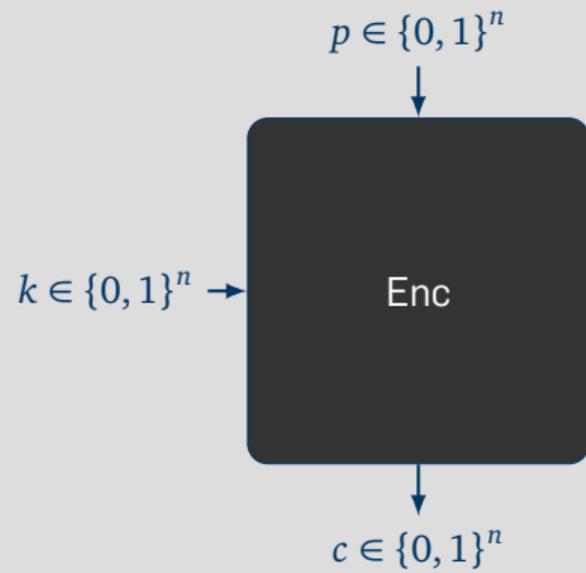
A small thumbnail version of the slide content is shown in the top right corner. It includes the title 'BISON Instantiating the Whitened Swap-Or-Not Construction', the date '2019-05-18', and the 'Two Parts' section header. The list of topics is also visible in a smaller font.

Overview
Topics covered

Two Parts

- Why do we need so many encryption rounds?
- Security argument for differential cryptanalysis.

Encrypt plaintext in blocks p of n bits,
under a key of n bits:



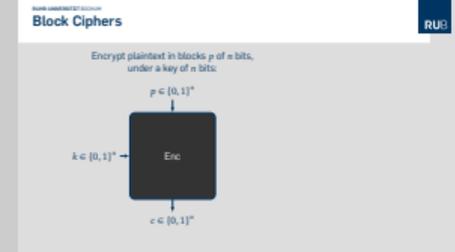
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BISON Instantiating the Whitened Swap-Or-Not

Construction

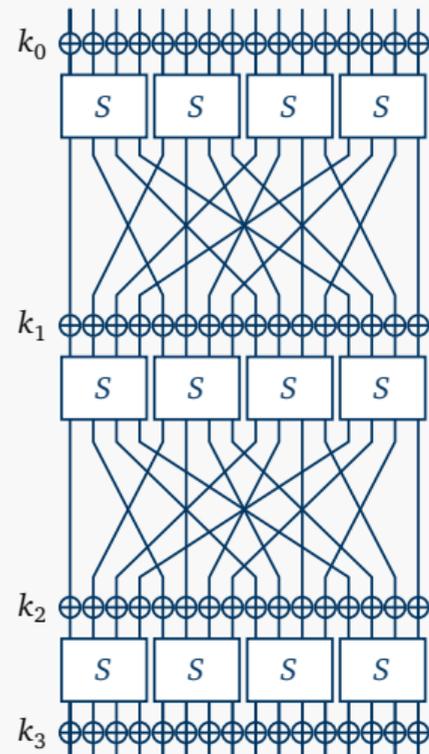
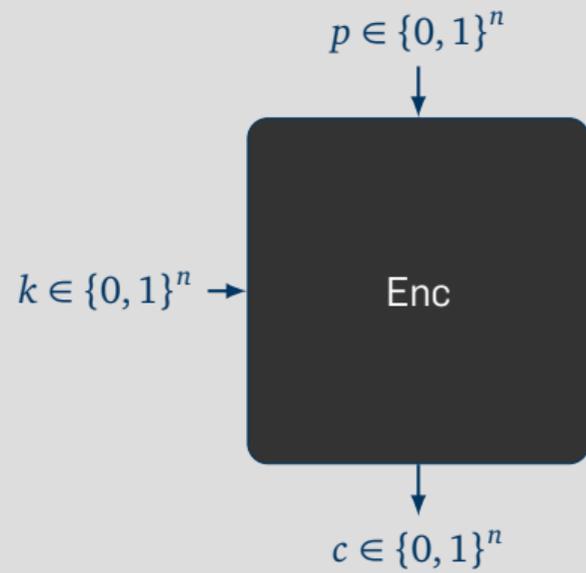
└ The WSN construction

└ Block Ciphers



- Block ciphers encrypt *blocks* of n -bit inputs under an n -bit master key
- As a basic cryptographic primitive, we need special modes of operations, if the data to be encrypted is not of exactly n -bit length.
- This we do not consider here, instead we want to look at how to build this black box.
- Typicall approach is an SPN structure, where key-addition, S-box layer and a linear layer are iterated over several rounds.
- Relatively well understood
- Good security arguments against known attacks
- There are some problems: differentials and linear hull effects

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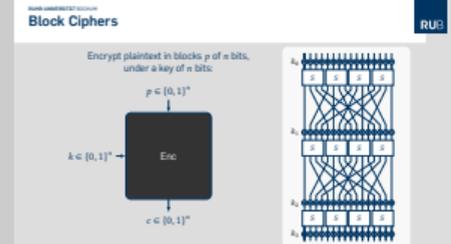
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BISON Instantiating the Whitened Swap-Or-Not

Construction

└ The WSN construction

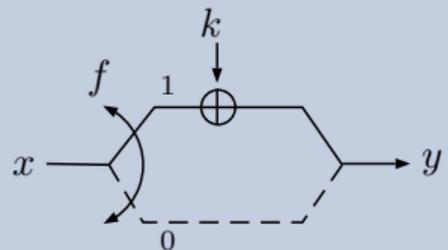
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- Relatively well understood
- Good security arguments against known attacks
- There are some problems: differentials and linear hull effects

Published by Tessaro at AsiaCrypt 2015 [ia.cr/2015/868].

Overview round, iterated r times



Whitened Swap-Or-Not round function

$$x, k \in \{0, 1\}^n \quad \text{and} \quad f_k : \{0, 1\}^n \rightarrow \{0, 1\}$$

$$y = \begin{cases} x + k & \text{if } f_k(x) = 1 \\ x & \text{if } f_k(x) = 0 \end{cases}$$

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└ The WSN construction

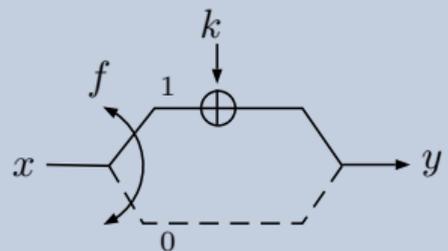
└ The WSN construction



- Lets take a look at the WSN construction (simplified).
- Again, an iterated round function, where the input is fed into from the left.
- Next, a Boolean function decides if either the round key k is xored onto the input, or nothing happens.
- The result is the updated state, respective the output of the round.
- In other words, x , and k are both n -bit strings and f is an n -bit Boolean function.
- The round output y is either $x + k$ if $f_k(x) = 1$ or just x in the other case.
- So why is this nice?

Published by Tessaro at AsiaCrypt 2015 [ia.cr/2015/868].

Overview round, iterated r times



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Security Proposition (informal)

The WSN construction with $r = \Theta(n)$ rounds is *Full Domain* secure.

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- The round output y is either $x + k$ if $f_k(x) = 1$ or just x in the other case.
- So why is this nice?
- Tessaro was able to show that this construction, when iterated over $\Theta(n)$ rounds, achieves *Full Domain* security (what ever that means).

- Sounds all very great.
- So from a practitioners point of view the natural next point is: lets implement it.

Construction

- $f_k(x) := ?$
- Key schedule?
- $\Theta(n)$ rounds?

Theoretical vs. practical constructions



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Construction

└ The WSN construction

└ An Implementation

An Implementation

Construction

- $f_k(x) := ?$
- Key schedule?
- $\Theta(n)$ rounds?

Theoretical vs. practical constructions



- Sounds all very great.
- So from a practitioners point of view the natural next point is: lets implement it.
- But ugh...
- How does this Boolean function f_k actually looks like?
- What about a key schedule? How do we derive the round keys?
- And how many are $\Theta(n)$ rounds?
- So, from a theoretical point of view we have a nice construction.
- But from a practical point of view it is basically useless.
- OK, let us fix this.

Input

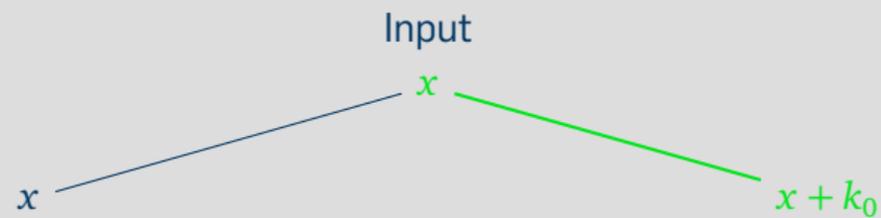
x

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Construction
└ Generic Analysis
└ The WSN construction



- We can observe an interesting first property, when looking at the encryption procedure round by round
- Starting with the plaintext x ...



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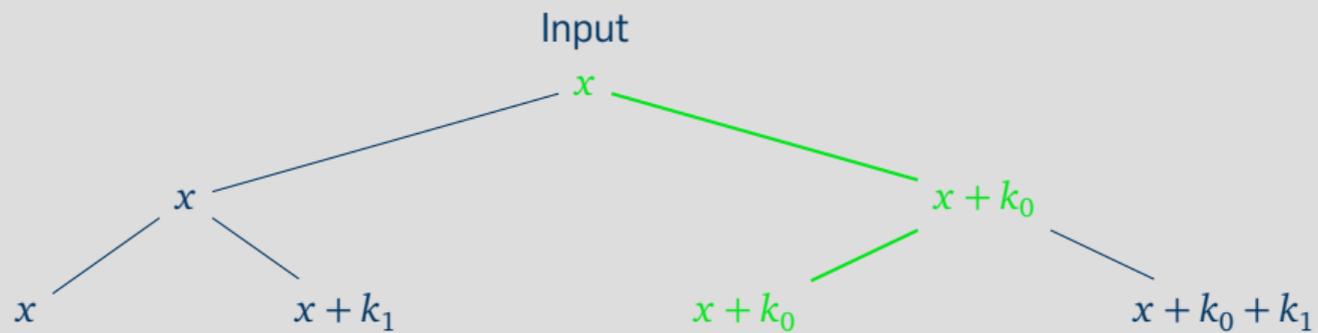
Construction

└ Generic Analysis

└ The WSN construction

The WSN construction
Encryption

- We can observe an interesting first property, when looking at the encryption procedure round by round
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- ...in each round, we either add the round key k_i , ...



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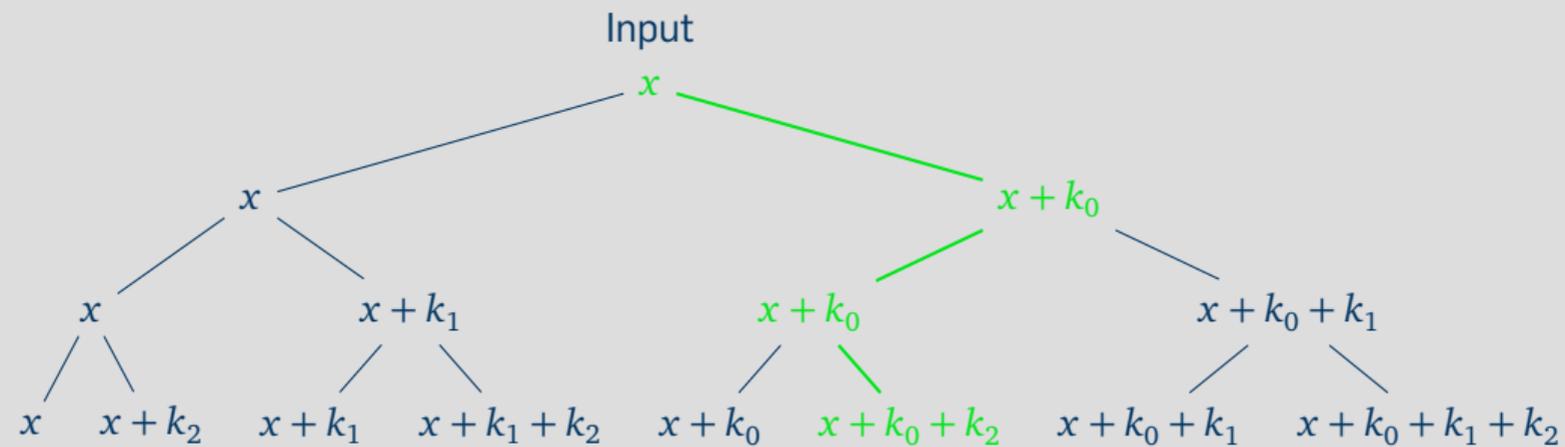
Construction

└ Generic Analysis

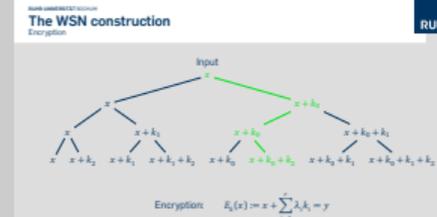
└ The WSN construction



- We can observe an interesting first property, when looking at the encryption procedure round by round
- Starting with the plaintext x ...
- ...in each round, we either add the round key k_i , ...
- ...or not.



Encryption: $E_k(x) := x + \sum_{i=1}^r \lambda_i k_i = y$



- We can observe an interesting first property, when looking at the encryption procedure round by round
- Starting with the plaintext $x \dots$
- ... in each round, we either add the round key k_i, \dots
- ... or not.
- Thus we end up with a binary tree of possible states.
- Furthermore, the encryption can also be written as the plaintext plus the sum of some round keys, chosen by the λ_i 's here.

Observation

- The ciphertext is the plaintext plus a subset of the round keys:

$$y = x + \sum_{i=1}^r \lambda_i k_i$$

- For pairs x_i, y_i : $\text{span} \{x_i + y_i\} \subseteq \text{span} \{k_j\}$.

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Construction

└ Generic Analysis

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Generic Analysis
On the number of rounds

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- First observation: $\text{span} \{x_i + y_i\} \subseteq \text{span} \{k_j\}$
- Leads to a simple distinguishing attack, *if number of rounds $r < n$.*

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Distinguishing Attack for $r < n$ rounds

There is an $u \in \mathbb{F}_2^n \setminus \{0\}$, s. t. $\langle u, x \rangle = \langle u, y \rangle$ holds always:

$$\begin{aligned} \langle u, y \rangle &= \left\langle u, x + \sum \lambda_i k_i \right\rangle \\ &= \langle u, x \rangle + \left\langle u, \sum \lambda_i k_i \right\rangle = \langle u, x \rangle + 0 \end{aligned}$$

for all $u \in \text{span} \{k_1, \dots, k_r\}^\perp \neq \{0\}$

Generic Analysis
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- First observation: $\text{span} \{x_i + y_i\} \subseteq \text{span} \{k_j\}$
- Leads to a simple distinguishing attack, if number of rounds $r < n$.
- It is easy to find a u , s. t. $\langle u, y \rangle = \langle u, x \rangle = 0$ for all $x, y = x, E(x)$.
- Simply use the bilinearity of the scalar product.
- Then any u from the dual space spanned by the round keys fullfills the above equation.
- As long as there are less than n round keys, this dual space has dimension greater or equal then one.

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Rationale 1

Any instance must iterate at least n rounds; any set of n consecutive keys should be linearly indep.

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- Simply use the bilinearity of the scalar product.
- Then any u from the dual space spanned by the round keys fullfills the above equation.
- As long as there are less than n round keys, this dual space has dimension greater or equal than one.
- A first design rationale is thus...

A bit out of the clear blue sky, but:

Rationale 2

For any instance, f_k has to depend on all bits, and for any $\delta \in \mathbb{F}_2^n$: $\Pr[f_k(x) = f_k(x + \delta)] \approx \frac{1}{2}$.

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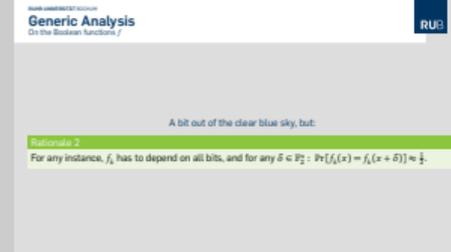
Construction

└ Generic Analysis

└ Generic Analysis

- We also need this second rationale.
- Its not so easy explainable without going into more depth.
- So you have to believe me on this one.
- It basically says that for any input difference $\delta \neq k$:

$$\Pr[f_k(x) = f_k(x + \delta)] \approx \frac{1}{2}$$



Generic Analysis
On the Boolean functions f

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A genus of the WSN family: BISON

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Generic properties of Bent whitened Swap Or Not (BISON)

- At least n iterations of the round function
- Consecutive round keys linearly independent
- The round function depends on all bits
- $\forall \delta : \Pr[f_k(x) = f_k(x + \delta)] = \frac{1}{2}$ (*bent*)

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BISON Instantiating the Whitened Swap-Or-Not Construction

Generic Analysis

A genus of the WSN family: BISON

- A quick recap and implications for any WSN instance.
- Rationale 2 basically tells us, we have to use *bent* functions.
- That's nice, as those functions are quite well understood and already well scrutinised.
- Also, this is the reason for the name: Bent Whitened Swap-Or-Not
- But, and that's not so nice...

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- Rationale 1**
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Rational 1 & 2: WSN is *slow* in practice!

The advantage?

Differential Cryptanalysis!

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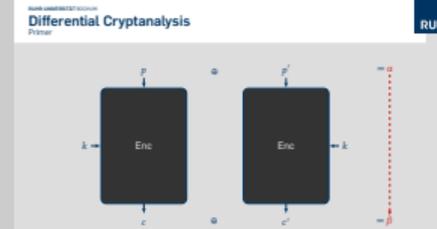
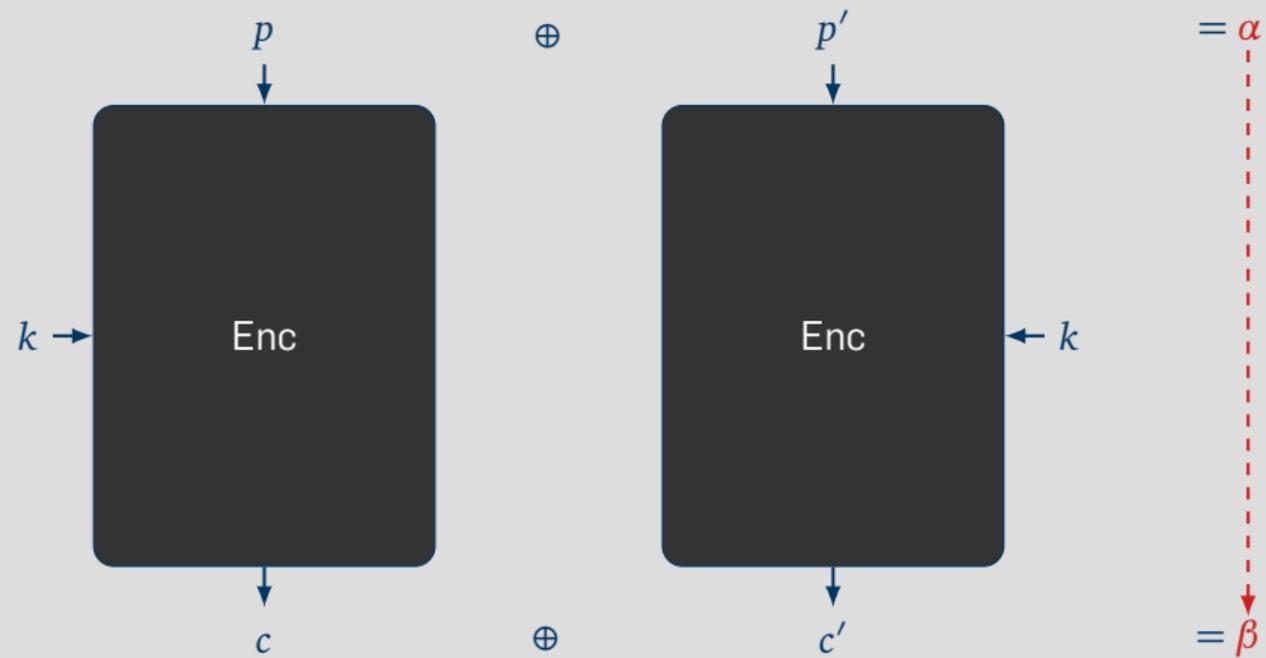
- Generic Analysis

└ A genus of the WSN family: BISON

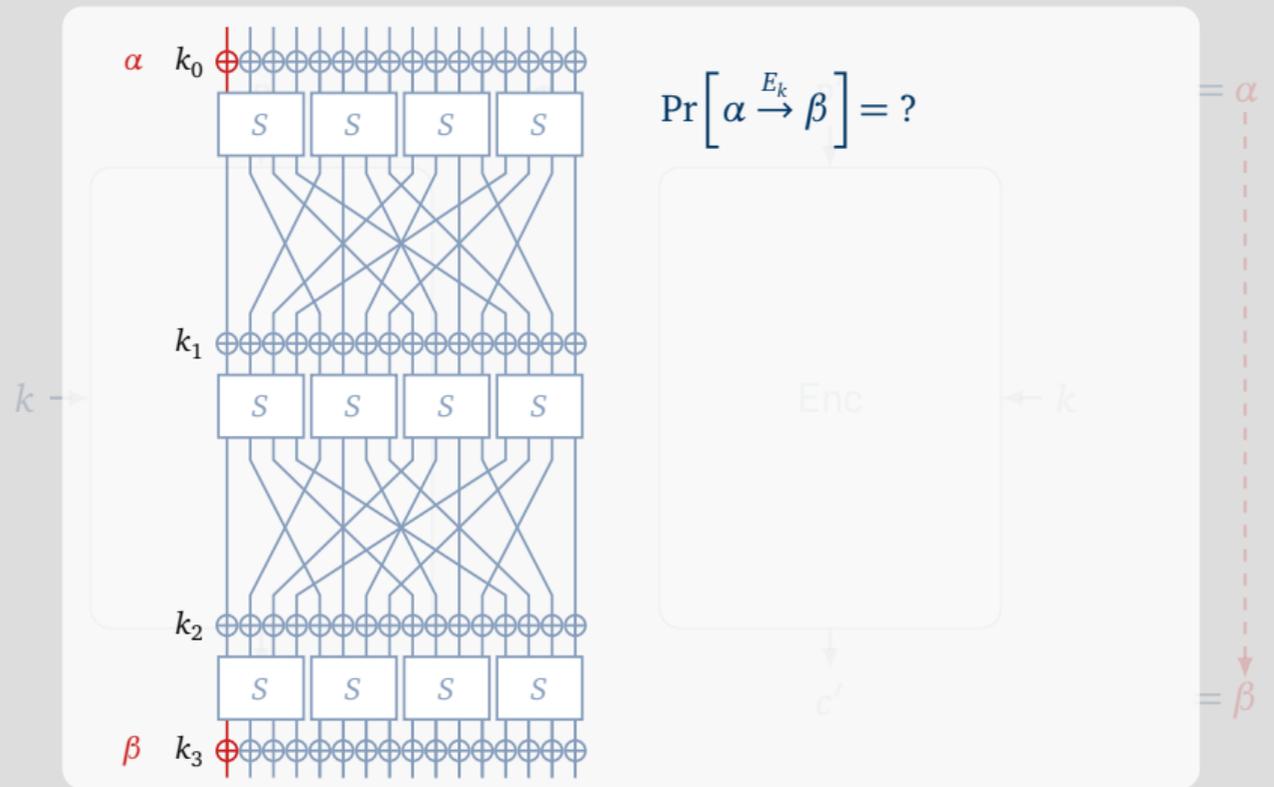
The screenshot shows a slide with the following content:

- Rationale 1:** Any instance must iterate at least n rounds; any set of n consecutive keys should be linearly indep.
- Rationale 2:** For any instance, f_k has to depend on all bits, and for any $\delta \in \mathbb{F}_2^n$: $\Pr[f_k(x) = f_k(x + \delta)] \approx \frac{1}{2}$.
- Generic properties of Bent whitened Swap Or Not (BISON):**
 - At least n iterations of the round function
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 - The round function depends on all bits
 - $\forall \delta : \Pr[f_k(x) = f_k(x + \delta)] = \frac{1}{2}$ (*bent*)
- The advantage?** Differential Cryptanalysis!
- Rational 1 & 2: WSN is slow in practice!**

- A quick recap and implications for any WSN instance.
- Rationale 2 basically tells us, we have to use *bent* functions.
- That's nice, as those functions are quite well understood and already well scrutinised.
- Also, this is the reason for the name: Bent Whitened Swap-Or-Not
- But, and that's not so nice...
- n iterations of a round function that depends on *all* bits will be slow
- Let me repeat this (Reviewer 2 argued that we should optimise more):
No matter how good we will optimise this: *it will be slow*
- For example, assume you can do one round in one clock cycle, this is still an order of magnitude slower than AES.
- So, why should we care about any instance?
- All hope is not lost, let's have a look at differential cryptanalysis!



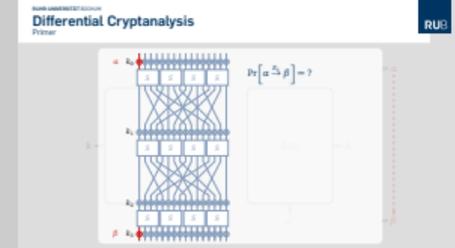
- For differential cryptanalysis, interested in propagation of input difference α to output difference β .
- Doing this in general at this abstraction level is a very hard problem.



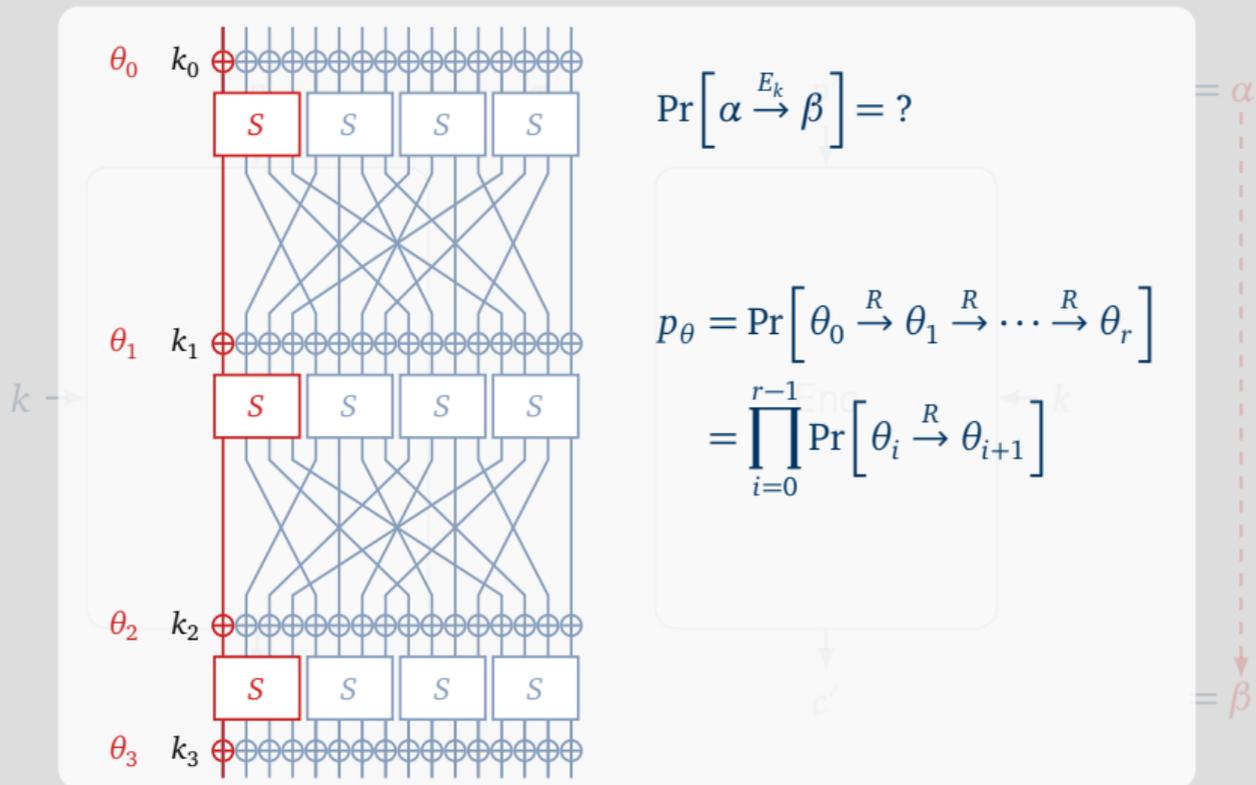
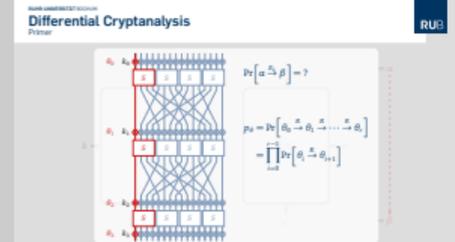
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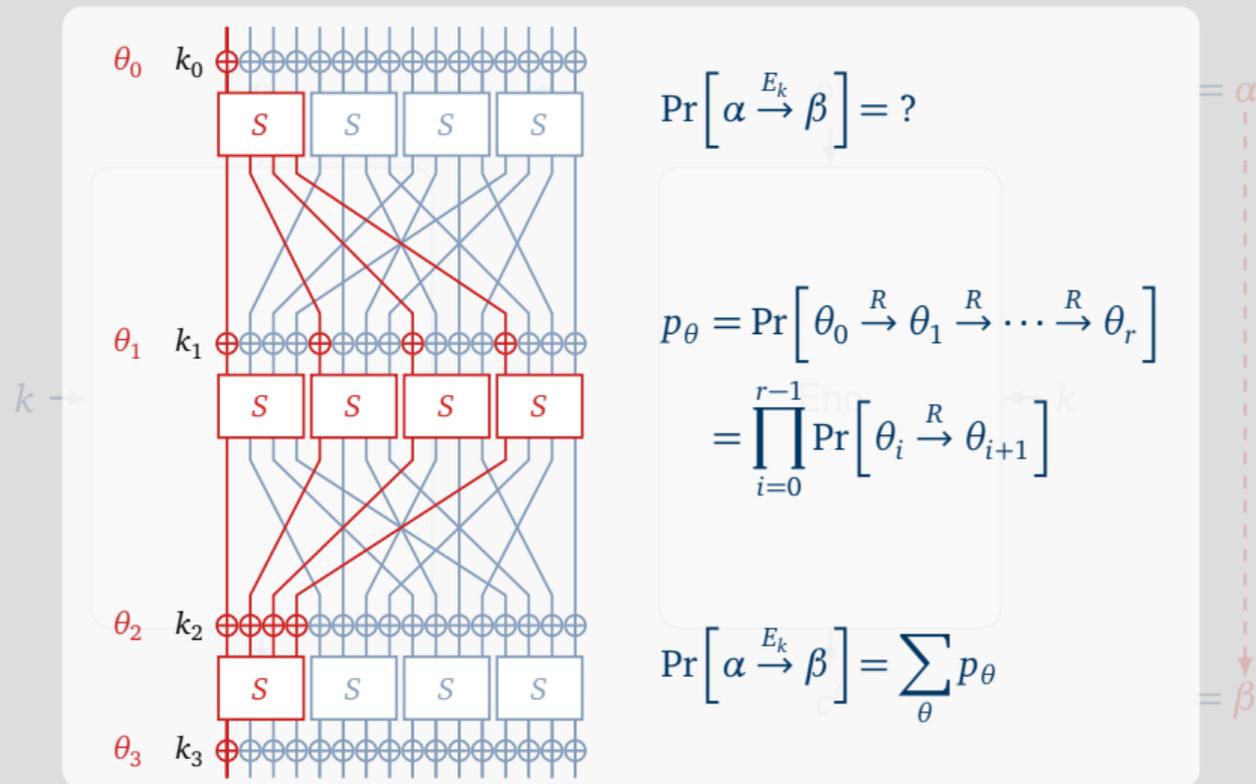
- Construction
 - └ Differential Analysis
 - └ Differential Cryptanalysis



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- To say anything, we usually look for single so called *trails* through the inner building blocks.
- Now, computing the probability of one such trail is actually doable.
- But, trails can go several alternative ways through non-linear parts, thus we have to cope with a branching effect...



$$\Pr[\alpha \xrightarrow{E_k} \beta] = ?$$

$$p_\theta = \Pr[\theta_0 \xrightarrow{R} \theta_1 \xrightarrow{R} \dots \xrightarrow{R} \theta_r]$$

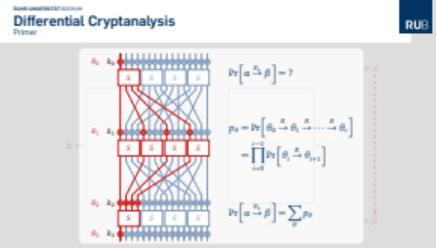
$$= \prod_{i=0}^{r-1} \Pr[\theta_i \xrightarrow{R} \theta_{i+1}]$$

$$\Pr[\alpha \xrightarrow{E_k} \beta] = \sum_{\theta} p_\theta$$

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BISON Instantiating the Whitened Swap-Or-Not Construction

- Construction
 - └ Differential Analysis
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- To say anything, we usually look for single so called *trails* through the inner building blocks.
- Now, computing the probability of one such trail is actually doable.
- But, trails can go several alternative ways through non-linear parts, thus we have to cope with a branching effect...
- And eventually, several of these trails cluster in a so called *differential*.
- While in this example it is still feasible, computing the *exact* probability in a real cipher is not.
- We thus have to restrain on bounding or approximating this probability.
- In the end, tight bounds for differentials over several rounds remain an open (but important!) problem.
- For BISON our aim is to give exactly this: a tight bound for any differential over several rounds.

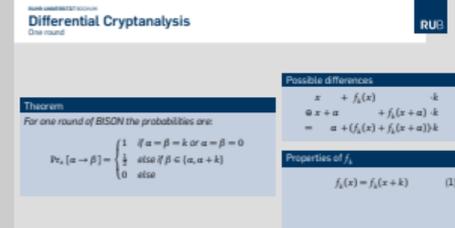
$$\Pr_x[\alpha \rightarrow \beta] = \begin{cases} 1 & \text{if } \alpha = \beta = k \text{ or } \alpha = \beta = 0 \\ \frac{1}{2} & \text{else if } \beta \in \{\alpha, \alpha + k\} \\ 0 & \text{else} \end{cases}$$

Theorem

For one round of BISON the probabilities are:

$$\Pr_x[\alpha \rightarrow \beta] = \begin{cases} 1 & \text{if } \alpha = \beta = k \text{ or } \alpha = \beta = 0 \\ \frac{1}{2} & \text{else if } \beta \in \{\alpha, \alpha + k\} \\ 0 & \text{else} \end{cases}$$

- We start by understanding the differential one round behaviour.
- For the three possible cases, let us look at what differences are actually possible.



Theorem

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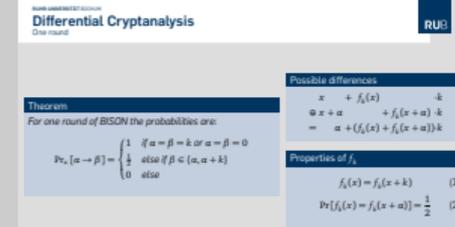
Possible differences

$$\begin{aligned} & x + f_k(x) \cdot k \\ \oplus & x + \alpha + f_k(x + \alpha) \cdot k \\ = & \alpha + (f_k(x) + f_k(x + \alpha)) \cdot k \end{aligned}$$

Properties of f_k

$$f_k(x) = f_k(x + k) \quad (1)$$

- We start by understanding the differential one round behaviour.
- For the three possible cases, let us look at what differences are actually possible.
- Remember that one round computes the output as $x + f_k(x) \cdot k$.
- With the input difference α we get as possible output differences $\beta \in \{0, \alpha, k, \alpha + k\}$.
- For decryption we need that x and $x + k$ are mapped to the same value by f_k
- Thus, $\beta = \alpha$ with probability one, if and only if $\alpha = k$ or $\alpha = 0$
- If β is not one of the above four values, such an input/output pair cannot occur, thus the probability is zero.



Theorem

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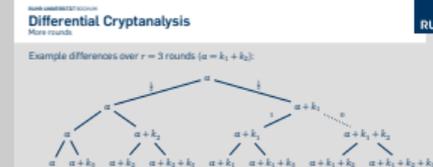
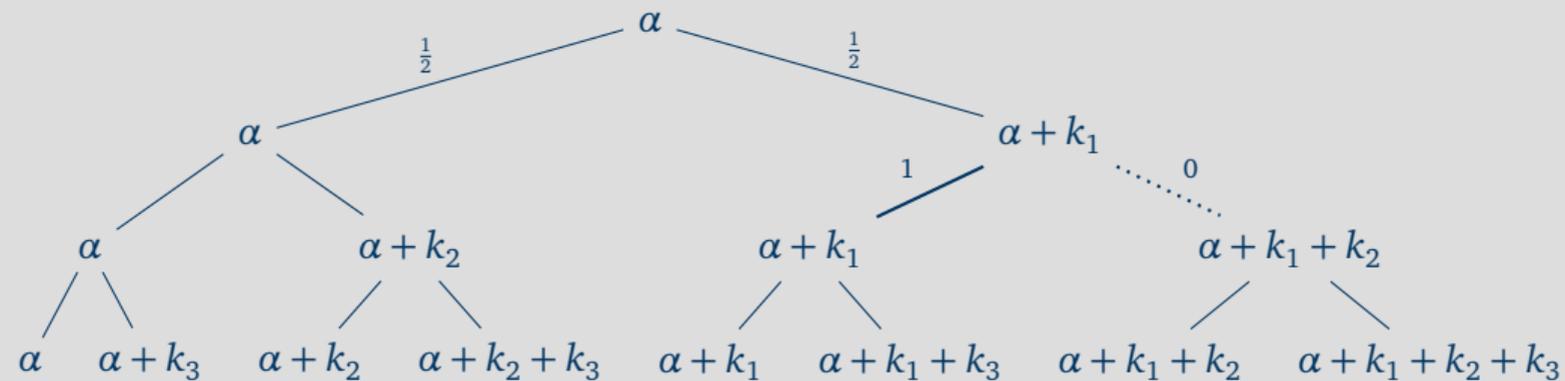
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Properties of f_k

$$\begin{aligned} f_k(x) &= f_k(x + k) & (1) \\ \Pr[f_k(x) = f_k(x + \alpha)] &= \frac{1}{2} & (2) \end{aligned}$$

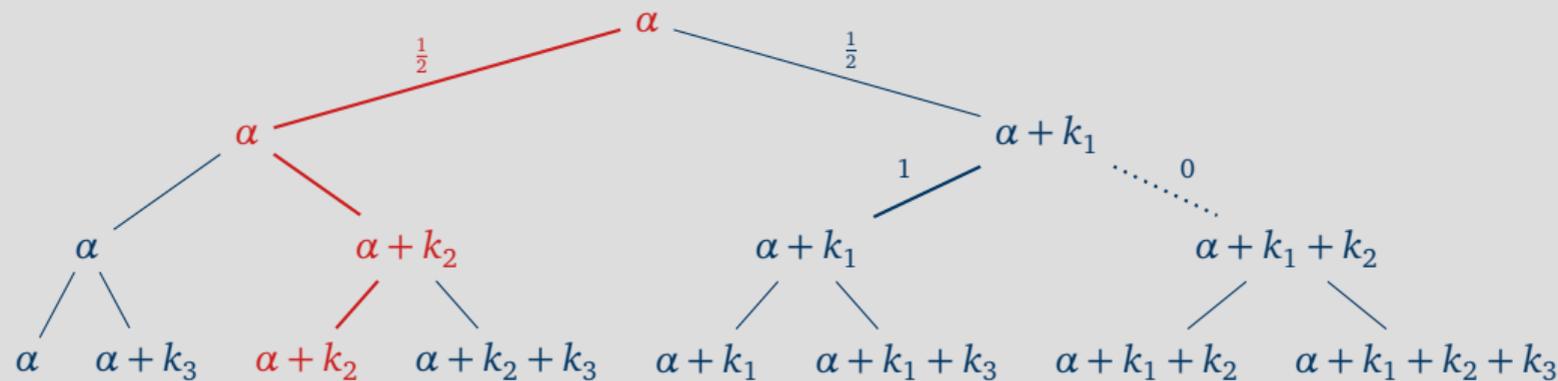
- We start by understanding the differential one round behaviour.
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- Thus, $\beta = \alpha$ with probability one, if and only if $\alpha = k$ or $\alpha = 0$
- If β is not one of the above four values, such an input/output pair cannot occur, thus the probability is zero.
- For the last case, remember that for any input difference, we required that f_k collides with probability one half.

Example differences over $r = 3$ rounds ($\alpha = k_1 + k_2$):



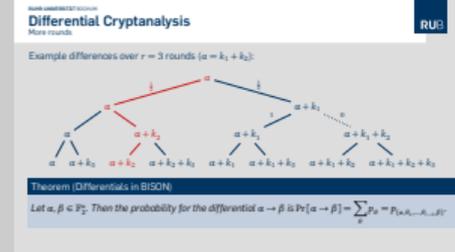
- When we look at more rounds, we can depict these cases again in this tree structure.
- Starting with the input difference α , choosing an input value x determines the path we take through the tree.
- Now assuming that $\alpha = k_1 + k_2$ it can not happen that the differential characteristic takes twice the right branch.
- This would result in a zero difference, which is not possible for permutations as long as the input difference is non-zero.

Example differences over $r = 3$ rounds ($\alpha = k_1 + k_2$):



Theorem (Differentials in BISON)

Let $\alpha, \beta \in \mathbb{F}_2^n$. Then the probability for the differential $\alpha \rightarrow \beta$ is $\Pr[\alpha \rightarrow \beta] = \sum_{\theta} P_{\theta} = P_{(\alpha, \theta_1, \dots, \theta_{r-1}, \beta)}$.



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- Now assuming that $\alpha = k_1 + k_2$ it can not happen that the differential characteristic takes twice the right branch.
- This would result in a zero difference, which is not possible for permutations as long as the input difference is non-zero.
- Regarding differentials, the important observation is:
- For any input/output pair (α, β) , there is only one path.
- In other words: no branching occurs and the differential consists of a single trail only.

A concrete species



2019-05-18

BISON Instantiating the Whitened Swap-Or-Not

Construction

└ The concrete Instance

└ BISON

BISON

A concrete species



- Up to now we do not have specified much more then the initial WSN construction had.
- For a concrete implementation, we still need to define
 - Number of rounds
 - Key Schedule
 - Boolean function f_k
- So let us look at a concrete BISON species
- In particular, we discuss how to tackle Rationales 1 and 2.

Rationale 1

Any instance must iterate at least n rounds; any set of n consecutive keys should be linearly indep.

Design Decisions

- Choose number of rounds as $3 \cdot n$
- Round keys derived from the state of LFSRs
- Add round constants c_i to w_i round keys

Implications

- Clocking an LFSR is cheap
- For an LFSR with irreducible feedback polynomial of degree n , every n consecutive states are linearly independent
- Round constants avoid structural weaknesses

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BISON Instantiating the Whitened Swap-Or-Not

Construction

└ The concrete Instance

└ Addressing Rationale 1

Addressing Rationale 1
The Key Schedule

Rationale 1
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Design Decisions

- Choose number of rounds as $3 \cdot n$
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Implications

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- Due to analysis of the alg. deg., we chose $3n$ rounds, see last slide.
- Deriving round keys from the states of LFSRs is efficiently implementable and fullfills our requirements for linear independent round keys.
- For those of you how attended Gregors talk at FSE:
 - While generating test vectors for the implementation we again noted some unwanted symmetries for encryptions with low hamming weight.
 - Thus we added round constants, to avoid these structural weaknesses.
 - (Sorry for fixing another cipher)

Rationale 2

For any instance, the f_k should depend on all bits, and for any $\delta \in \mathbb{F}_2^n$: $\Pr[f_k(x) = f_k(x + \delta)] \approx \frac{1}{2}$.

Design Decisions

- Choose $f_k : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$ s. t.

$$\delta \in \mathbb{F}_2^n : \Pr[f_k(x) = f_k(x + \delta)] = \frac{1}{2},$$

that is, f_k is a bent function.

- Choose the simplest bent function known:

$$f_k(x, y) := \langle x, y \rangle$$

Implications

- Bent functions well studied
- Bent functions only exists for even n
- Instance not possible for every block length n

└ The concrete Instance

└ Addressing Rationale 2

The thumbnail slide contains the following text:

Rationale 2
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Implications

- Bent functions well studied
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- Instance not possible for every block length n

- Just chose the simplest bent function, the scalar product.
- This is also efficiently implementable.
- But, another drawback:
- Bent functions exists only for even n .
- Thus BISON cannot be instantiated for every block length n .
- In particular, due to reasons not covered here, we can actually only instantiate it for *odd* block lengths.

Construction

- $f_k(x) := ?$
- Key schedule?
- $\Theta(n)$ rounds?



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BISON Instantiating the Whitened Swap-Or-Not

Construction

└ The concrete Instance

└ An Implementation

An Implementation

Construction

- $f_k(x) := ?$
- Play schedule?
- $\Theta(n)$ rounds?



- Coming back to our initial question.
- And basically only for the sake of completeness, as we already saw this is going to be slow.

Construction

- $f_k(x, y) := \langle x, y \rangle$
- Key schedule: LFSRs.
- $\Theta(n) = 3n$ rounds.

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BISON Instantiating the Whitened Swap-Or-Not

Construction

└ The concrete Instance

└ An Implementation

An Implementation

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- We have specified everything, so let's benchmark against AES (what else).

Construction

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* AES-128 on Skylake Intel® Core i7-7800X @ 3.5GHz, see Daemen et al. [The design of Xoodoo and Xoofff, Table 5].

† BISON on CoffeeLake Intel® Core i7-8700 @ 3.7 GHz.

Cipher	Block size (bit)	Cycles/Byte mean
AES*	128	0.65
BISON†	129	3 064.08

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BISON Instantiating the Whitened Swap-Or-Not

Construction

└ The concrete Instance

└ An Implementation

- Coming back to our initial question.
- And basically only for the sake of completeness, as we already saw this is going to be slow.
- We have specified everything, so let's benchmark against AES (what else).
- OK, told you so, BISON is like 4 700 times slower than AES.
- Or: more than three orders of magnitude.
- Optimising this will not help enough.

An Implementation

Construction

- $f_k(x, y) := \langle x, y \rangle$
- Key schedule: LFSRs.
- $\Theta(n) = 3n$ rounds.

Cipher	Block size (bit)	Cycles/Byte mean
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Linear Cryptanalysis

For $r \geq n$ rounds, the correlation of any non-trivial linear trail for BISON is upper bounded by $2^{-\frac{n+1}{2}}$.

Zero Correlation

For $r > 2n - 2$ rounds, BISON does not exhibit any zero correlation linear hulls.

Algebraic Degree and Division Property

Algebraic degree grows *linearly*. Conservative estimate: for $r \geq 3n$ rounds, no attack possible.

Invariant Attacks

For $r \geq n$ rounds, neither invariant subspaces nor nonlinear invariant attacks do exist for BISON.

Impossible Differentials

For $r > n$ rounds, there are no impossible differentials for BISON.

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BISON Instantiating the Whitened Swap-Or-Not

Construction

└ Further Analysis

└ Further Cryptanalysis

Further Cryptanalysis	
Linear Cryptanalysis For $r \geq n$ rounds, the correlation of any non-trivial linear trail for BISON is upper bounded by $2^{-\frac{n+1}{2}}$.	Invariant Attacks For $r \geq n$ rounds, neither invariant subspaces nor nonlinear invariant attacks do exist for BISON.
Zero Correlation For $r > 2n - 2$ rounds, BISON does not exhibit any zero correlation linear hulls.	Impossible Differentials For $r > n$ rounds, there are no impossible differentials for BISON.
Algebraic Degree and Division Property Algebraic degree grows linearly. Conservative estimate: for $r \geq 3n$ rounds, no attack possible.	

- We did more cryptanalysis, but our results are more of the “classical” kind.
- For linear cryptanalysis, we bound the correlation of any non-trivial trail.
- Current known security arguments for resistance against invariant attacks apply.
- Zero correlation and impossible differentials do not exist for $2n$ rounds or more.
- Best attacks seem to exploit the algebraic degree.
- We show that it grows only linearly – which is especially bad in comparison to SPN ciphers.
- The result on the algebraic degree also applies to NLFSRs or maximally unbalanced Feistel networks.
- Conservative estimation: might work for more than $2n$ rounds, but not for $3n$ or more.

Conclusion/Questions

Thank you for your attention!

BISON

- A first instance of the WSN construction
- Good results for differential cryptanalysis

Open Problems

- Construction for linear cryptanalysis?
- Similar args. for Unbalanced Feistel?

Thank you!

Questions?

2018/1011

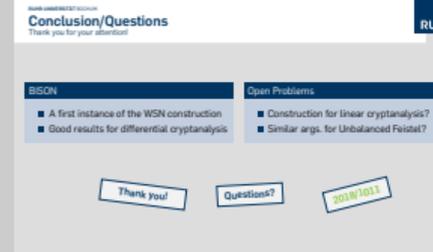
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BISON Instantiating the Whitened Swap-Or-Not

Construction

└ Further Analysis

└ Conclusion/Questions



2019-05-18
BISON Instantiating the Whitened Swap-Or-Not
Construction
└─ Further Analysis

[Details](#)

Details

BISON's round function

For round keys $k_i \in \mathbb{F}_2^n$ and $w_i \in \mathbb{F}_2^{n-1}$ the round function computes

$$R_{k_i, w_i}(x) := x + f_{b(i)}(w_i + \Phi_{k_i}(x)) \cdot k_i.$$

where

- Φ_{k_i} and $f_{b(i)}$ are defined as

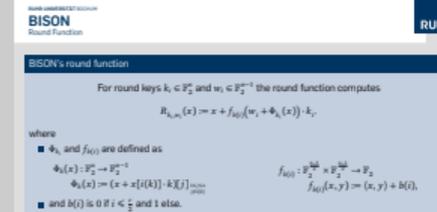
$$\Phi_k(x) : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^{n-1}$$

$$\Phi_k(x) := (x + x[i(k)] \cdot k)[j]_{\substack{1 \leq j \leq n \\ j \neq i(k)}}$$

$$f_{b(i)} : \mathbb{F}_2^{\frac{n-1}{2}} \times \mathbb{F}_2^{\frac{n-1}{2}} \rightarrow \mathbb{F}_2$$

$$f_{b(i)}(x, y) := \langle x, y \rangle + b(i),$$

- and $b(i)$ is 0 if $i \leq \frac{r}{2}$ and 1 else.



BISON's key schedule

Given

- primitive $p_k, p_w \in \mathbb{F}_2[x]$ with degrees $n, n - 1$ and companion matrices C_k, C_w .
- master key $K = (k, w) \in (\mathbb{F}_2^n \times \mathbb{F}_2^{n-1}) \setminus \{0, 0\}$

The i th round keys are computed by

$$\text{KS}_i : \mathbb{F}_2^n \times \mathbb{F}_2^{n-1} \rightarrow \mathbb{F}_2^n \times \mathbb{F}_2^{n-1}$$

$$\text{KS}_i(k, w) := (k_i, c_i + w_i)$$

where

$$k_i = (C_k)^i k, \quad c_i = (C_w)^{-i} e_1, \quad w_i = (C_w)^i w.$$

BISON Key Schedule

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