The Invariant Set Attack

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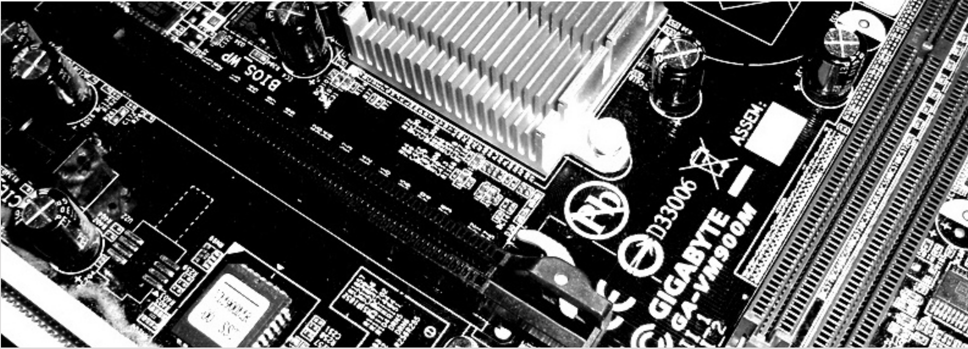


Nonlinear Invariant Attack

Practical Attack on Full SCREAM, iSCREAM, and Midori64

Paper

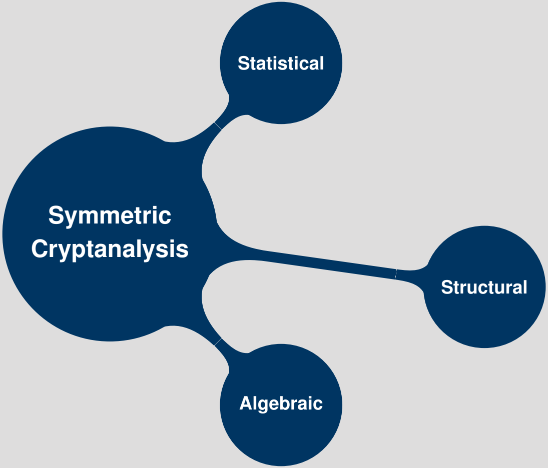
- Todo, Leander, and Sasaki [TLS16] at AsiaCrypt'16
- Structural attack, breaks SCREAM, iSCREAM and Midori64 (surprise, surprise)[1] in the *weak key setting*

Organisation

The paper in 50 words:

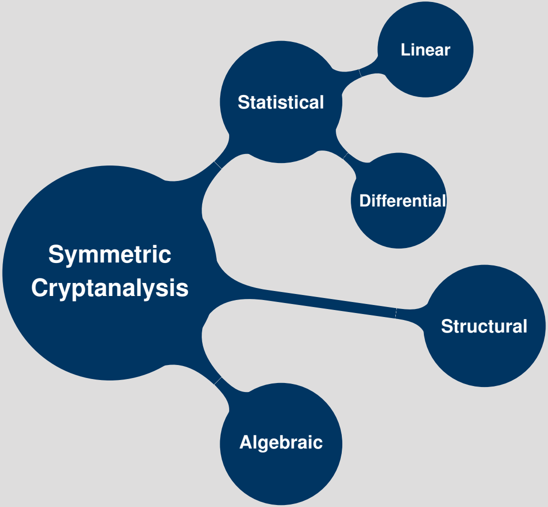
[1] Useless LaTeX Fact: Did you know that `\time` is an anagram of `\item`?

Context
or: similar attacks?



Context

or: similar attacks?



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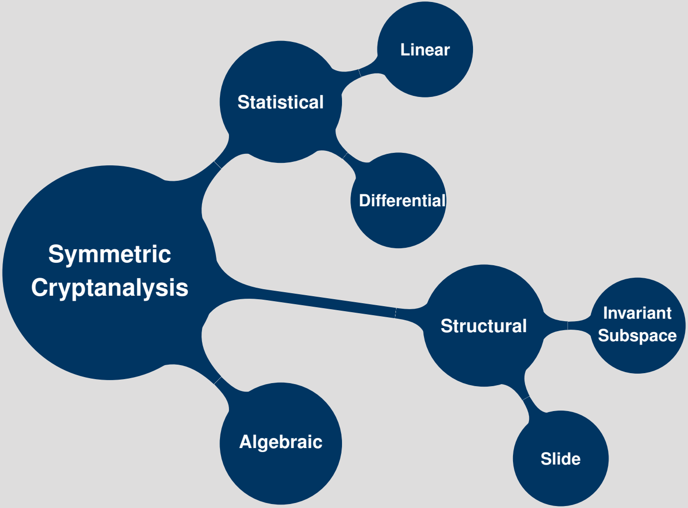

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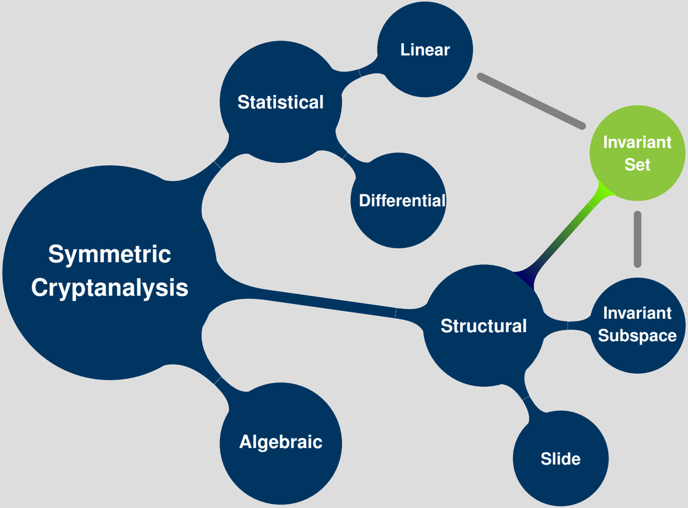

Linear Cryptanalysis

Taking the fun out of it

- invented by Matsui [Mat93]
- broke DES
- together with Differential Cryptanalysis best studied attack on block ciphers

Image: `http://www.isce2009.ryukoku.ac.jp/eng/keynote_address.html`

Linear Cryptanalysis

Taking the fun out of it

Core Idea

Given a block cipher $E_k : \mathbb{F}_2^n \to \mathbb{F}_2^n$, find an *input mask* $\alpha \in \mathbb{F}_2^n$ and an *output mask* $\beta \in \mathbb{F}_2^n$, s. t.

$$\langle \alpha, x \rangle \oplus \langle \beta, E_k(x) \rangle = c$$

holds with high probability for a constant c.

- $\alpha \xrightarrow{E_k} \beta$ is called a *linear approximation* of E_k
- much more to deal with: we have to keep the distribution over k in mind and so on and so forth

Invariant Subspace Attack

Almost there

- invented by Leander *et al.* [Lea+11]
- broke PRINTCIPHER

Illustration

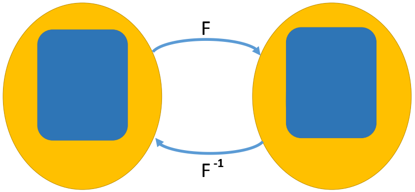

Image: `http://www.lightsec.org/2013/images/gregor_leander.jpg`

Invariant Subspace Attack

Almost there

RUB

Core Idea

Given a block cipher $E_k : \mathbb{F}_2^n \to \mathbb{F}_2^n$, s. t. $E_k(x) = E(x \oplus k)$, assume that there exists a subspace $U \subseteq \mathbb{F}_2^n$, s. t.

$$E(U \oplus c) = U \oplus d$$

for two constants c, d.

- A key $k = u \oplus c \oplus d$ is called *weak*, if $u \in U$.
- For a weak key:
  $E_k(U \oplus d) = E((U \oplus d) \oplus (u \oplus c \oplus d)) = E(U \oplus c) = U \oplus d.$
- We thus can distinguish encryptions under a weak key.

Invariant Set Attack

or: Nonlinear Invariant Attack

RUB

Core Idea

Given a block cipher $E_k : \mathbb{F}_2^n \to \mathbb{F}_2^n$, s. t. $E_k(x) = E(x \oplus k)$, find an efficiently computable nonlinear Boolean function $g : \mathbb{F}_2^n \to \mathbb{F}_2$, s. t.

$$g(E(x \oplus k)) = g(x \oplus k) \oplus c = g(x) \oplus g(k) \oplus c \tag{1}$$

for a constant c and many k.

- g is called *nonlinear invariant*
- keys for which Eq (1) holds are called *weak keys*

Invariant Set Attack

Step-by-Step

Typical block cipher construction: key-alternating function

Let $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ and $E_{k_1,k_2,\dots,k_r} : \mathbb{F}_2^n \to \mathbb{F}_2^n$ be of the form

$$E_k(x) = F(\cdots F(x \oplus k_1) \cdots \oplus k_r).$$

Invariant Set Attack

Step-by-Step

Notation: we write $y_0 = x$, $y_i = F(y_{i-1} \oplus k_i)$, and thus $y_r = E_k(x)$.

Nonlinear invariant for the round function

Assume there exists a nonlinear invariant g for F, s. t. all keys k_i are weak. Then:

Invariant Set Attack

Step-by-Step

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Nonlinear invariant for the round function

Assume there exists a nonlinear invariant g for F, s. t. all keys k_i are weak. Then:

$$g(E_k(x)) = g(y_r)$$

Invariant Set Attack

Step-by-Step

Notation: we write $y_0 = x$, $y_i = F(y_{i-1} \oplus k_i)$, and thus $y_r = E_k(x)$.

Nonlinear invariant for the round function

Assume there exists a nonlinear invariant g for F, s. t. all keys k_i are weak. Then:

$$\begin{aligned} g(E_k(x)) &= g(y_r) \\ &= g(F(y_{r-1} \oplus k_r)) \end{aligned}$$

Invariant Set Attack

Step-by-Step

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$$\begin{aligned} g(E_k(x)) &= g(y_r) \\ &= g(F(y_{r-1} \oplus k_r)) \\ &= g(y_{r-1}) \oplus g(k_r) \oplus c_r \end{aligned}$$

Invariant Set Attack

Step-by-Step

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Nonlinear invariant for the round function

Assume there exists a nonlinear invariant g for F, s. t. all keys k_i are weak. Then:

$$\begin{aligned} g(E_k(x)) &= g(y_r) \\ &= g(F(y_{r-1} \oplus k_r)) \\ &= g(y_{r-1}) \oplus g(k_r) \oplus c_r \\ &\vdots \\ &= g(x) \oplus \bigoplus_{i=1}^{r} g(k_i) \oplus c_1 \end{aligned}$$

Invariant Set Attack

Weak Keys

It seems quite unlikely that Eq (1) holds for many k?

Example nonlinear invariant

$$g : \mathbb{F}_2^4 \to \mathbb{F}_2$$
$$(x_4, x_3, x_2, x_1) \mapsto x_4 x_3 \oplus x_3 \oplus x_2 \oplus x_1$$

Invariant Set Attack

Weak Keys

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Example nonlinear invariant

$$g : \mathbb{F}_2^4 \to \mathbb{F}_2$$
$$(x_4, x_3, x_2, x_1) \mapsto x_4 x_3 \oplus x_3 \oplus x_2 \oplus x_1$$

g is nonlinear invariant for key xor and has 4 weak keys:

Split g in a nonlinear part f and a linear part ℓ:

$$g(x_4, x_3, x_2, x_1) = f(x_4, x_3) \oplus \ell(x_2, x_1)$$

All k of the form $k = (0, 0, k_2, k_1)$ are weak – and these are exactly four possible keys.

Results

RUB

Attack Complexities

	# Weak k	max. # Recovered Bits
SCREAM	2^{96}	32 bits
iSCREAM	2^{96}	32 bits
Midori64	2^{64}	$32h$ bits

	Data Complexity	Time Complexity
SCREAM	33 ciphertexts	32^3
iSCREAM	33 ciphertexts	32^3
Midori64	$33h$ ciphertexts	$32^3 \cdot h$

Questions?

Thank you for your attention!

Mainboard & Questionmark Images: flickr

References I

[Lea+11] G. Leander, M. A. Abdelraheem, H. AlKhzaimi, and E. Zenner. “A Cryptanalysis of PRINTcipher: The Invariant Subspace Attack”. In: *CRYPTO*. Vol. 6841. LNCS. Springer, 2011, pp. 206–221.

[Mat93] M. Matsui. “Linear Cryptanalysis Method for DES Cipher”. In: *EUROCRYPT*. Vol. 765. LNCS. Springer, 1993, pp. 386–397.

[TLS16] Y. Todo, G. Leander, and Y. Sasaki. “Nonlinear Invariant Attack - Practical Attack on Full SCREAM, iSCREAM, and Midori64”. In: *ASIACRYPT (2)*. Vol. 10032. LNCS. 2016, pp. 3–33.