RUHR-UNIVERSITÄT BOCHUM

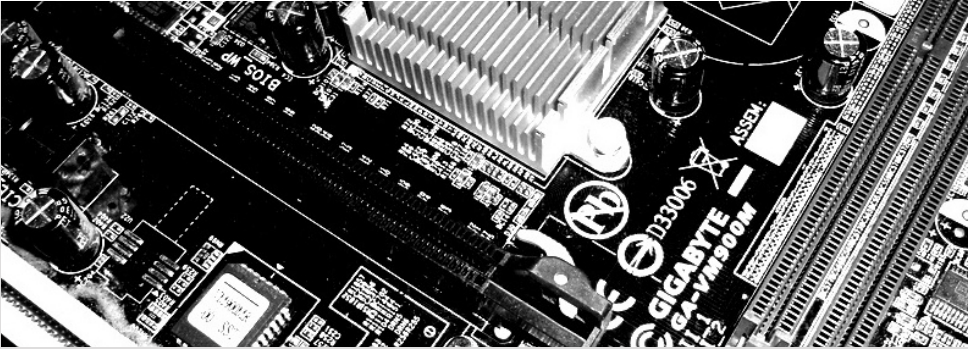
Attacks on Lattice Crypto

December 7th, 2016

FluxFingers

Workgroup Symmetric Cryptography
Ruhr University Bochum

Friedrich Wiemer



Why is Lattice Based Crypto important?

Or interesting? Or. . . ? Buzzword Bingo.

Some facts

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- It is damn fast (faster than dinosauRS cryptA)
- You can build anything you want from it:
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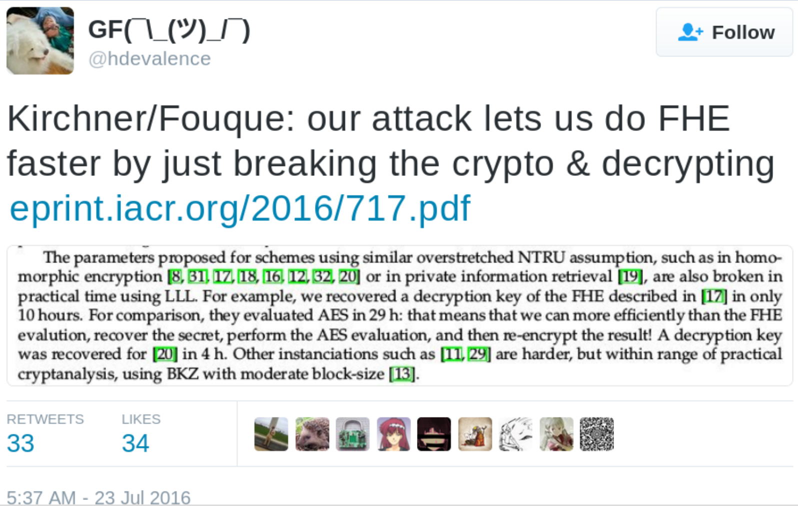
- It allows to build even some of the most advanced cryptographic building blocks:
  - Fully Homomorphic Encryption (FHE),
  - Multi-linear Maps,
  - Identity-based Encryption (IBE),
  - . . .

Why is Lattice Based Crypto important?

Is everything done?

Fully Homomorphic Encryption



GF(¯_(ツ)_/¯)
@hdevalence

Kirchner/Fouque: our attack lets us do FHE faster by just breaking the crypto & decrypting eprint.iacr.org/2016/717.pdf

The parameters proposed for schemes using similar overstretched NTRU assumption, such as in homomorphic encryption [8, 31, 17, 18, 16, 12, 32, 20] or in private information retrieval [19], are also broken in practical time using LLL. For example, we recovered a decryption key of the FHE described in [17] in only 10 hours. For comparison, they evaluated AES in 29 h: that means that we can more efficiently than the FHE evalution, recover the secret, perform the AES evaluation, and then re-encrypt the result! A decryption key was recovered for [20] in 4 h. Other instanciations such as [11, 29] are harder, but within range of practical cryptanalysis, using BKZ with moderate block-size [13].

RETWEETS 33 LIKES 34

5:37 AM - 23 Jul 2016

The new cool kid in town.

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- “Lattice based Crypto is one of the most promising PQC candidates blablabla” (almost every paper on lattices)

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- Google even implemented this in Chrome [Goob]
- So, research is really vibrant here

Everything was fine. And then Shor entered the stage...

A cryptographic thriller

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- . . . and published an efficient CVP quantum algorithm [ES16]
- for one day the cryptographic community was shocked!

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- ...and published an efficient CVP quantum algorithm [ES16]
- for one day the cryptographic community was shocked!
- ...and then Regev saved us all by finding a flaw in the paper [Reg]
- but still, Google stopped its PQ key exchange experiment with New Hope [Gooa]

Enough motivation!

How does Lattice Crypto work?

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Wait! Lattice, wtf?

Definition:

A lattice L is an discrete, additive, abelian subgroup of $\mathbb{R}^n$.

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Let $b_1, b_2, \ldots, b_d \in \mathbb{R}^n,\ d \leqslant n$ linear independent. Then the set

$$\mathrm{L} = \left\{ v \in \mathbb{R}^n \;\middle|\; v = \sum_{i=1}^{d} a_i b_i, a_i \in \mathbb{Z} \right\}$$

is a lattice.

Hey! You promised, this will be easy!

Lattice, dt.: Gitter

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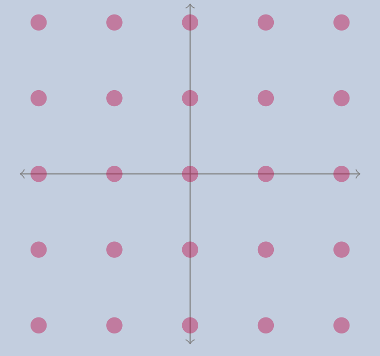
OK, OK, we can say it easier: $\mathbb{Z}^2$ is a Lattice

Example lattice



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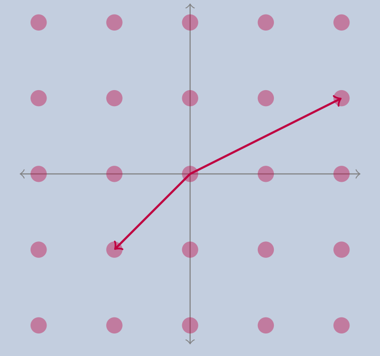
Random Basis



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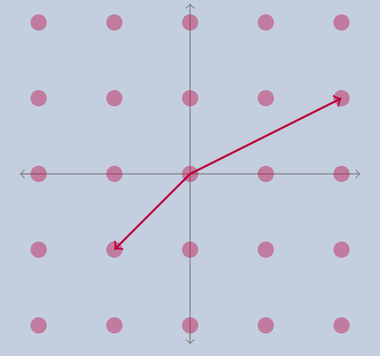
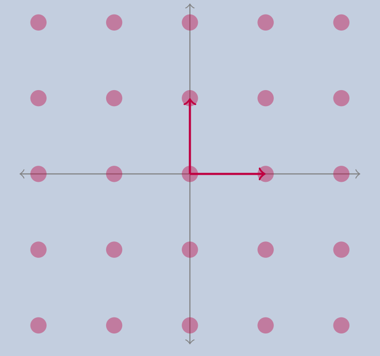


Reduced Basis



In general, basis reduction is a hard problem! The LLL and BKZ algorithm are available for this. NTL's implementation of BKZ has 2^{n^2} runtime.

Hard Problems in Lattices…

…are what we need for crypto.

Shortest Vector Problem (SVP)

Given a lattice L, what is a shortest vector $v \in \mathrm{L} \setminus \{0\}$?

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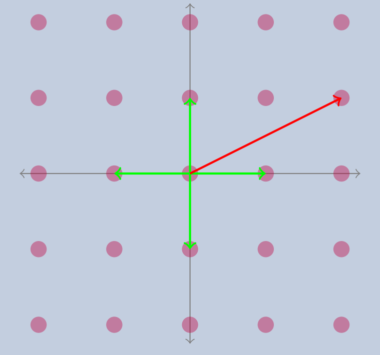

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Closest Vector Problem (CVP)

Given a lattice L and a target $\mathrm{t} \notin \mathrm{L}$, what is the closest vector $v \in \mathrm{L}$ to t?

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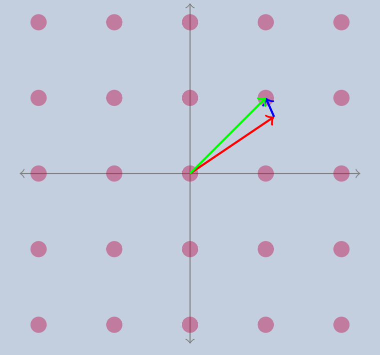
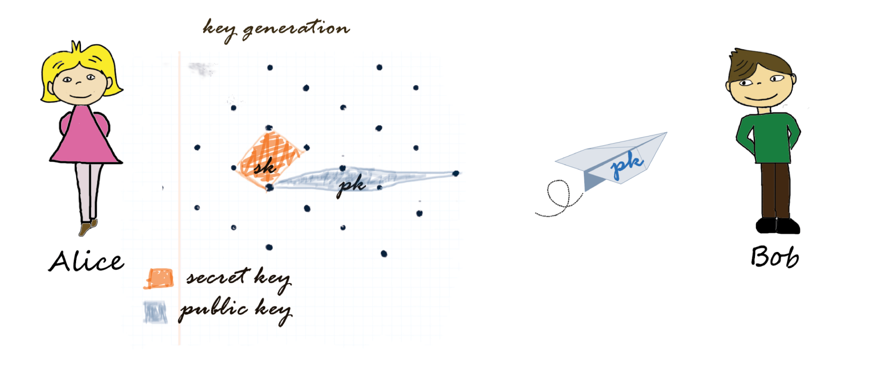


Lattice Based Crypto

Learning With Errors – or: the equivalent to textbook RSA

Key Generation[1]



[1] Thanks to Elena for the nice pictures.

Lattice Based Crypto

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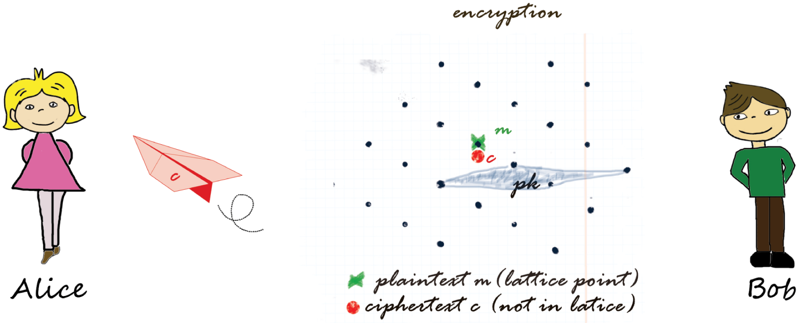
Encryption



Lattice Based Crypto

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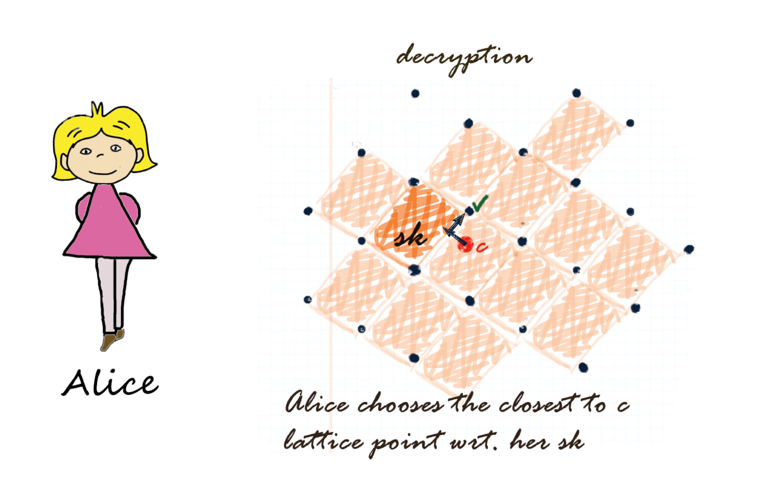
Decryption



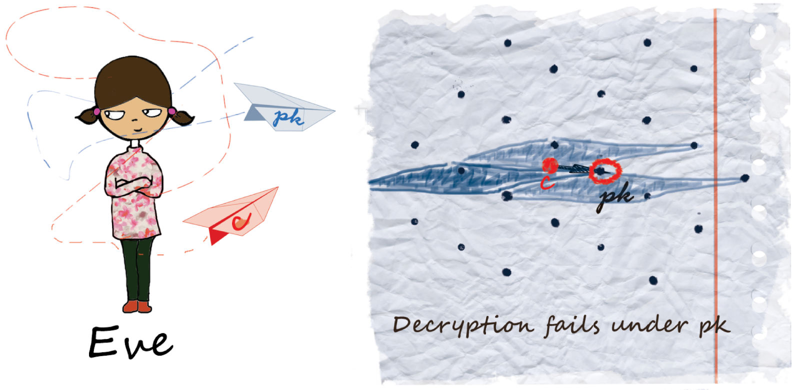
Attack Algorithm

In practice most efficient strategy is Babai's Nearest Plane [Bab86], improved by Lindner and Peikert [LP11] and Gama *et al*. [GNR10].

Nearest Plane

or BDD Enumeration

Attack

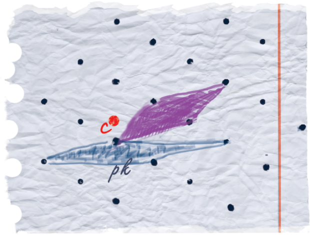


Nearest Plane

or BDD Enumeration

Step 1: Basis Reduction

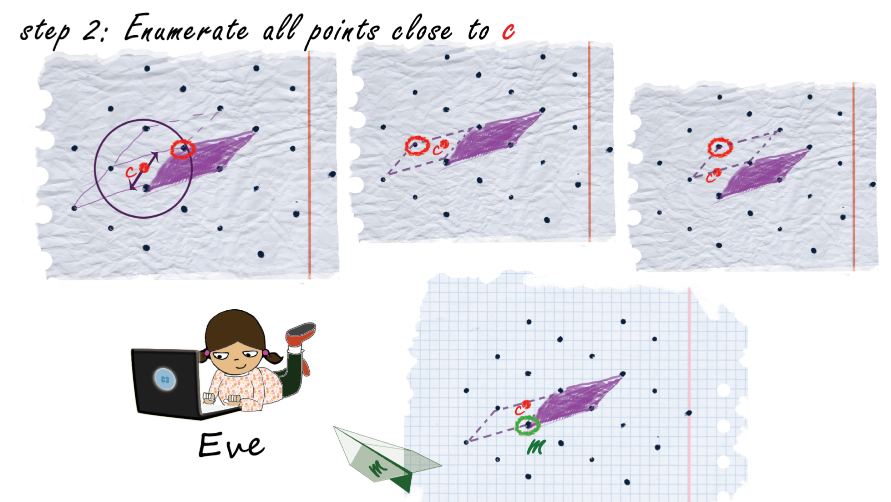
step1: Find an approximation to sk



Nearest Plane

or BDD Enumeration

Step 2: Enumerate Nearest Planes



Parallel Implementation of BDD enumeration for LWE

Finally, what we (joint work with Elena Kirshanova and Alex May) did:

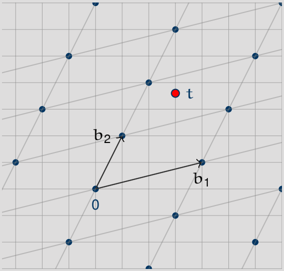
Research Project

- Goal: What is the *practical* runtime of BDD enumeration?
- Build a parallel implementation of `NearestPlanes`.
- Test this on some large scale parallel system.
- Hopefully break some real world parameters.

Parallelisation of Enumeration

Elena’s explanation

Closest point search via depth-first tree-traversal:



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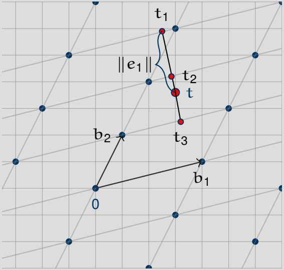

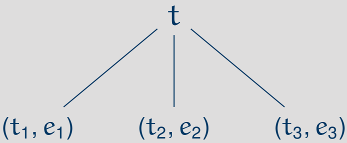

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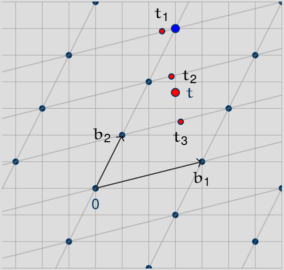

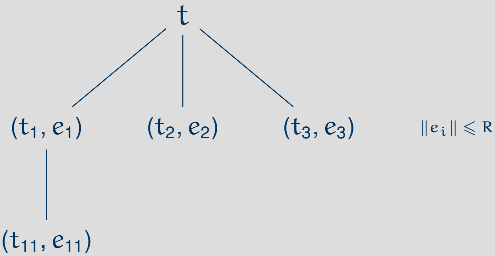

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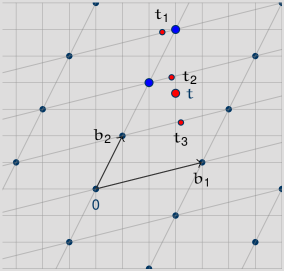

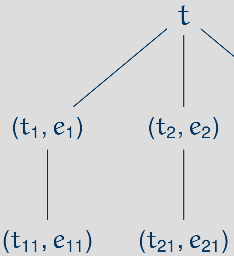

$\|e_i\| \leqslant R$

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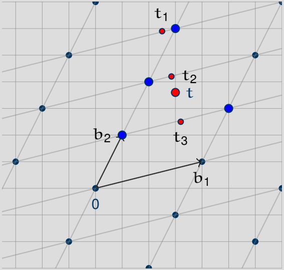

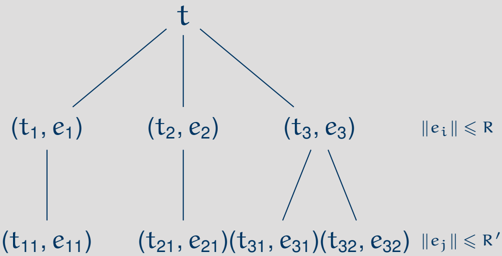
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(t_1, e_1) (t_2, e_2) (t_3, e_3) $\|e_i\| \leqslant R$

(t_{11}, e_{11}) $(t_{21}, e_{21})(t_{31}, e_{31})(t_{32}, e_{32})$ $\|e_j\| \leqslant R'$

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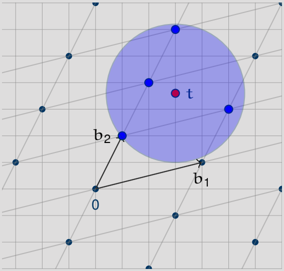

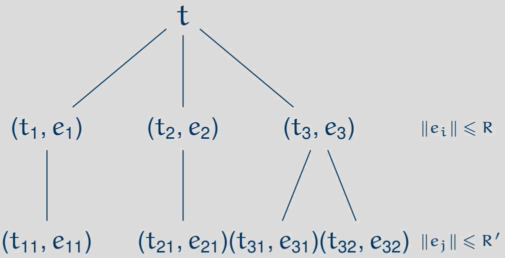

# Leaves to visit $= 2^{n \log n}$ for n-dim BDD

Results

After more than one year of work, two submissions and something like over 9000 weeks of benchmarking

We ended up with:

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We ended up with:

- an open source implementation:
  `https://github.com/pfasante/cvp-enum`
- an ACNS paper [KMW16] and a Best Student Paper Award 😃
- huge table of runtimes

Results: Numbers!

Standard LWE

LWE-parameters			BKZ-reduction	Enumeration	
n	q	$\lvert e \rvert \leqslant$	T	# Threads	T
90	4093	10	11.3h	1	35h
90	4093	10	11.3h	10	3.6h
100	4093	10	7h	24	2.7h

To be compared with: $(n = 192, \lvert e \rvert < 18, q = 4093)$ reaches 2^{87}-security level [LP11].

Results: Numbers!

LWE variant: Small secret

LWE-parameters			BKZ-reduction	Enumeration	
n	q	m	T	# Threads	T
140	16411	170	12h	1	16h
140	16411	170	12h	10	1.7h

To be compared with: $(n = 128, q = 16411, m = 2^{28}, T = 13\text{h})$ for combinatorial attack on LWE [KF15].

Results: Numbers!

LWE variant: Binary matrix

LWE-parameters n	 q	 m	BKZ-reduction T	Enumeration T
256	500009	440	4.5h	2min

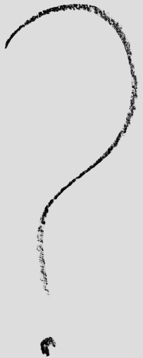
To be compared with: Estimation by Galbraith [Gal] roughly one day.

Questions?

Thank you for your attention!

Review

- Working as an engineer together with mathematicans can be fun
  You can code, they. . . can do math ☺
- Even if you don't understand what you are implementing, you can get something working out of it
- Eventually you'll understand the math 🥷



Mainboard & Questionmark Images: flickr

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